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Addendum to "Hyperbolic space cosmologies"

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Abstract: In our earlier paper [JHEP 0310 (2003) 058], we considered higher dimensional cosmological models with hyperbolic spaces. In particular the eternal accelerating expansion was obtained by studying small perturbation around the critical non-accelerated solution for $D > 10$. In this addendum, we show that there is also such a solution in the critical case $D = 10$.

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In our earlier paper [1], we considered higher dimensional cosmological models with hyperbolic spaces. In particular the eternal accelerating expansion was analyzed by studying small perturbations around the critical non-accelerated solution. We concluded that eternal acceleration is possible for $D > 10$. In this note we want to show that there is also such a solution in the case $D = 10$, which was overlooked in [1]. The existence of such a solution was recently argued in refs. [2, 3].

For this critical dimension, our metric is

$$ds^2 = \mathrm{e}^{-6\phi}\left(-dt^2 + a^2 ds^2_{H_3}\right) + \mathrm{e}^{2\phi} ds^2_{H_6}, \tag{1}$$

where $ds^2_{H_n}$ denotes the metric of an n-dimensional hyperbolic space. The critical solution is

$$a = 2t, \qquad \phi = \frac{1}{2\sqrt{6}}\psi + \frac{1}{8}\ln 30, \tag{2}$$

where

$$\psi = \frac{1}{c}\ln\left(\frac{t^2}{3}\right), \qquad c = 2\sqrt{\frac{2}{3}}. \tag{3}$$

Previously in our paper [1] we obtained accelerating solutions by perturbing the above solution as

$$a = a_0 + a_1, \qquad \psi = \psi_0 + \psi_1, \tag{4}$$

where a_0 and ψ_0 are the critical solutions in (2) and (3). The perturbative parts were written as

$$a_1 = A, \qquad \psi_1 = \frac{B}{t} \tag{5}$$

where a_1 and ψ_1 were chosen to be polynomials which were then fixed using the equations of motion. For perturbation theory to be valid, it was necessary to assume that $a_1 \ll a_0$ and $\psi_1 \ll \psi_0$. Our ansatz led to solutions with accelerated expansion for generic $D > 10$. For the case $D = 10$ the analysis resulted in constant A and B, and therefore there was no eternal acceleration. However, we now re-examine this case and perform more detailed analysis for the case $D = 10$ allowing the possibility that A and B depend on time in search of accelerated expansion.

Our approach is to re-examine the equations of motion for $D = 10$ and directly look for solutions starting from the above ansatz with arbitrary functions A and B. The equations of motion for the first order term (given as eqs. (5.11)-(5.13) in [1]) reduce in this particular case to

$$\dot{A} - \frac{3A}{4t} - \frac{c}{4}\dot{B} + \frac{3c}{4}\frac{B}{t} = 0, \tag{6}$$

$$\ddot{B} + \frac{\dot{B}}{t} + \frac{3B}{t^2} + \frac{3}{c}\left(\frac{\dot{A}}{t} - \frac{A}{t^2}\right) = 0, \tag{7}$$

$$\ddot{A} + \frac{c}{t}\dot{B} = 0. \tag{8}$$

To obtain a solution we first eliminate A from eqs. (6) and (7) and get as a result

$$\dot{A} - ct\ddot{B} - 2c\dot{B} = 0, \tag{9}$$

where the dot denotes the derivative with respect to time. If we differentiate this result once and use eq. (8) to eliminate the variable A, we obtain the equation for B:

$$t^2 \dddot{B} + 3t\ddot{B} + \dot{B} = 0, \tag{10}$$

whose solution is

$$B = c_1(\ln t)^2 + c_2 \ln t + c_3. \tag{11}$$

Finally we obtain the solution for the variable A by substituting the solution for B into eq. (9)

$$A = c\left[c_1(\ln t)^2 + (2c_1 + c_2)\ln t + c_4\right]. \tag{12}$$

Consistency of the equations tells us that

$$c_4 = \frac{8}{3}c_1 + c_2 + c_3. \tag{13}$$

The expansion and acceleration of the cosmic evolution are

$$\dot{a} = 2 + ct^{-1}(2c_1 \ln t + 2c_1 + c_2), \quad \ddot{a} = -ct^{-2}(2c_1 \ln t + c_2). \tag{14}$$

The acceleration occurs for all

$$2c_1 \ln t + c_2 < 0, \qquad \text{or} \qquad t > \mathrm{e}^{-c_2/2c_1}, \tag{15}$$

if we choose $c_1 < 0$. The latter condition is necessary for the accelerated expansion to continue in the future direction. Since the expanding condition, which is also the condition for the perturbation theory to be valid,

$$t > -\frac{c}{2}(2c_1 \ln t + 2c_1 + c_2), \tag{16}$$

can always be satisfied for sufficiently large t, the accelerated expanding phase always occurs after certain time of evolution

Although a_1 approaches infinity as $t \to \infty$, the first order perturbative treatment above is still reliable for large t because $\ln t/t \ll 1$ and we still have $a_1 \ll a_0$ and $\psi_1 \ll \psi_0$ for all t. Thus hyperbolic compactification can give eternal acceleration for $D \geq 10$ instead of just $D > 10$.

As a final comment, our solution is related to eq. (3.19) in [2] via the time redefinition

$$t = \left(\frac{5}{8}\right)^{3/2} \frac{1}{4}\left[\tau^4 - 18c_0(\ln \tau)^2 - 36d_0 \ln \tau\right], \tag{17}$$

where τ is the time coordinate and c_0, d_0 are parameters of the result in [2]. Here the factor 5/8 comes from the different conventions for hyperbolic space by a constant scale factor. Our seemingly extra parameter c_3 is a redundancy and can be absorbed by the coordinate transformation of scaling t.

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References

[1] C.M. Chen, P.M. Ho, I.P. Neupane, N. Ohta and J.E. Wang, *Hyperbolic space cosmologies*, *JHEP* **10** (2003) 058 [hep-th/0306291]. See also references therein.

[2] L. Andersson and J.M. Heinzle, *Eternal acceleration from M-theory*, hep-th/0602102.

[3] J. Sonner and P.K. Townsend, *Recurrent acceleration in Dilaton-Axion cosmology*, hep-th/0608068.